Sec. 11.2B Synthetic and Long Division

Dividing Polynomials Using Long Division

Quotient

Divisor $\longrightarrow 12\overline{)638} \longleftarrow$ Dividend

Remainder – what is left over
To Divide:

1. Divide the leading term by the leading term of the divisor. Enter the result over the radical sign.
2. Multiply the divisor by the result and enter below the dividend.
3. Subtract and bring down the remaining terms.
4. Repeat the first 3 steps until finished.

Ex: a. $4x^3 - 3x^2 + x + 1$ divided by $x + 2$

$$\begin{array}{r}
4x^2 - 11x + 23 \\
x+2\ \overline{)\,4x^3 - 3x^2 + x + 1} \\
-\ (4x^3 + 8x^2) \\
\hline
-11x^2 + x + 1 \\
-\ (-11x^2 - 22x) \\
\hline
23x + 1 \\
-\ (23x + 46) \\
\hline
-45
\end{array}$$

$$\boxed{4x^2 - 11x + 23 + \frac{-45}{x+2}}$$

b. $-2 + 3x^3 - x^2 + x$ divided by $x^2 + 2$

$$\begin{array}{r}
3x - 1 \\
x^2+2\ \overline{)\,3x^3 - x^2 + x - 2} \\
-\ (3x^3 \qquad + 6x) \\
\hline
-x^2 - 5x - 2 \\
-\ (-x^2 \qquad - 2) \\
\hline
-5x
\end{array}$$

$$\boxed{3x - 1 + \frac{-5x}{x^2+2}}$$

c. $4x - 3x^2 + x^5 - 5x^4 + 6$ divided by $2 + x^2$

$$\begin{array}{r}
x^3 - 5x^2 - 2x + 7 \\
x^2+2\ \overline{)\,x^5 - 5x^4 - 3x^2 + 4x + 6} \\
-\ (x^5 \qquad + 2x^3) \\
\hline
-5x^4 - 2x^3 - 3x^2 + 4x + 6 \\
-\ (-5x^4 \qquad - 10x^2) \\
\hline
-2x^3 + 7x^2 + 4x + 6 \\
-\ (-2x^3 \qquad - 4x) \\
\hline
7x^2 + 8x + 6 \\
-\ (7x^2 \qquad + 14) \\
\hline
8x - 8
\end{array}$$

$$\boxed{x^3 - 5x^2 - 2x + 7 + \frac{8x-8}{x^2+2}}$$

Dividing Polynomials Using Synthetic Division

To Divide:

1. Write the dividend in descending powers of *x*. Copy the coefficients, remembering to insert 0 for any missing powers of *x*.
2. Insert the usual division symbol. If the divisor is *x* - #, enter the # to the left of the division symbol. If it is *x* + #, enter -#.
3. Bring the first number down to the answer row.
4. Multiply the first entry by the divisor and place in row 2, 1 column to the right.
5. Add the last entry in row 2 to the entry above it in row 1 and enter in the answer in row 3.
6. Continue until finished.
7. The final entry in row 3 is your remainder. The other entries are the coefficients of x starting with the exponent whose degree is 1 less than you started with.

Ex: Divide $-4x^3+2x^2-x+1$ by $x+2$.

$$\begin{array}{r|rrrr} -2 & -4 & 2 & -1 & 1 \\ & & 8 & -20 & 42 \\ \hline & -4 & 10 & -21 & 43 \end{array}$$

$$\boxed{-4x^2+10x-21+\frac{43}{x+2}}$$

Ex: Divide $x^6-16x^4+x^2-16$ by $x+4$

$$\begin{array}{r|rrrrrrr} -4 & 1 & 0 & -16 & 0 & 1 & 0 & -16 \\ & & -4 & 16 & 0 & 0 & -4 & 16 \\ \hline & 1 & -4 & 0 & 0 & 1 & -4 & 0 \end{array}$$

$$\boxed{x^5-4x^4+x-4}$$

Ex: Show that g(x) = x + 3 is a factor of f(x) = $-4x^3+5x^2+8$.

$$\begin{array}{r|rrrr} -3 & -4 & 5 & 0 & 8 \\ & & 12 & -51 & 153 \\ \hline & -4 & 17 & -51 & 161 \end{array}$$

$$-4x^2+17x-51+\frac{161}{x+3}$$

NOT A FACTOR

REMAINDER WOULD BE 0.

Ex: If you know that *x* = -7 is one zero, use synthetic division and factoring to find the other zeros of the function x^3+4x^2-21x.

$$\begin{array}{r|rrrr} -7 & 1 & 4 & -21 & 0 \\ & & -7 & 21 & 0 \\ \hline & 1 & -3 & 0 & 0 \end{array}$$

$x^2-3x=0$

$x(x-3)=0$

$\boxed{x=0}$ $\quad x-3=0$

$\boxed{x=3}$

Homework: pg. 1028 #1, 3, 5, 12, 13, 15, 23, 31, 32, 33, 34, 39, 41, 42, 43, 44